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Effects of the fourth generation on $\Delta M_{B_{d,s}}$ in B^0–$\bar{B}^0$ mixing

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Abstract. We investigate the mass differences $\Delta M_{B_{d,s}}$ in $B^0_{d,s}$–$\bar{B}^0_{d,s}$ mixing with a new up-like quark t' in the sequential fourth generation model. We give the basic formulae for $\Delta M_{B_{d,s}}$ in this model and obtain two kinds of numerical results for ΔM_{B_s} as a function of $m_{t'}$. We find that one of our results can satisfy the present experimental lower bound of ΔM_{B_s}. We also get the constraints of the fourth generation CKM factor $V^*_{t'b}V_{t'd}$, as a function of t', from the experimental measurements of ΔM_{B_d}. Thus, $\Delta M_{B_{d,s}}$ provides a possible test of the fourth generation and may give a signal of new physics.

1 Introduction

The standard model (SM) is a very successful theory of the elementary particles known today. But it must be incomplete because it has too many unpredicted parameters (*nineteen!*) to be put in by hand. Most of these parameters are in the fermion part of the theory. We do not know the source of the quarks and leptons, nor how to obtain their mass and number theoretically. We have to get information of them all from experiment.

From the point of view of phenomenology, for fermions, there is a realistic question as to the number of the fermion generations, or whether there are other additional quarks or leptons. The present experiments tell us there are only three generations of fermions with *light* neutrinos with masses smaller than $M_Z/2$ [1]. But the experiments do not exclude the existence of other additional generations, such as the fourth generation, with a *heavy* neutrino, i.e. $m_{\nu_4} \geq M_Z/2$ [2]. Many authors have studied models which extend the fermion part, such as vector-like quark models [3], sterile neutrino models [4] and the sequential fourth generation standard model (SM4) [5] which we study in this note. We consider a sequential fourth generation non-SUSY model [5], to which is added an up-like quark t', a down-like quark b', a lepton τ', and a heavy neutrino ν' in the SM. The properties of these new fermions are all the same as their corresponding counterparts of the other three generations except their masses and CKM mixing; see Table 1.

Table 1. The elementary particle spectrum of SM4

	up-like quark	down-like quark	charged lepton	neutral lepton
SM fermions	u	d	e	ν_e
	c	s	μ	ν_μ
	t	b	τ	ν_τ
new fermions	t'	b'	τ'	$\nu_{\tau'}$

There are a lot of references about the fourth generation [5–8]. In our previous papers [7,8], we investigated the rare B-meson decays with the fourth generation [7] and ϵ'/ϵ in the K^0 systems in SM4 [8]. We got some interesting results, such as new effects of the fourth generation particles on the meson decays and CP violation. We also got the constraints of the fourth generation CKM matrix factors, like $V^*_{t's}V_{t'}b$ from $B \to X_s\gamma$ [7] and $V^*_{t's}V_{t'}d$ from ϵ'/ϵ [8]. In other words, these rare decays provide possible tests of the existence of a fourth generation.

In this note, we study the mass difference $\Delta M_{B_{d,s}}$ in the B^0–$\bar{B}^0$ system [10,11] with a fourth generation. We will give the prediction of ΔM_{B_s} in SM4 and obtain the constraints of the new fourth generation CKM matrix factor $V^*_{t'b}V_{t'}d$ from ΔM_{B_d}. Particle–antiparticle mixings are responsible for the small mass differences between the mass eigenstates of the neutral mesons, such as ΔM_K in K_L–K_S mixing and $\Delta M_{B_{d,s}}$ in B^0–$\bar{B}^0$ mixing. Being FCNC processes, they involve heavy quarks in loops and consequently are perfect testing grounds for heavy flavor physics. For example, B^0–$\bar{B}^0$ mixing [12] gave the first indication of a large top quark mass. K_L–K_S mixing is also closely related to the violation of the CP symmetry which is experimentally known since 1964 [13]. These are sensitive measures of the top quark t couplings $V_{ti}(i = d, s, b)$ and of the top quark mass m_t. The experimental measurement of ΔM_{B_d} is used to determine the CKM matrix elements V_{td} [10]. It offers an improved determination of the unitarity triangle with the use of the future accurate measurement of ΔM_{B_s} [10,11]. For physics beyond the SM, there are a number of studies of such effects in B_d decays [14,11,15]. But the B_s system has received somewhat less attention from the new physics point of view [16, 11, 15]. Experimentally, ΔM_{B_d} has been accurately measured: $\Delta M_{B_d} = 0.473 \pm 0.016\,\mathrm{ps}^{-1}$ [11, 17]. But ΔM_{B_s} has only a lower bound: $\Delta M_{B_s} > 14.3\,\mathrm{ps}^{-1}$ [11, 18, 17].

In this note, we want to investigate $\Delta M_{B_{d,s}}$ in B^0–$\bar{B}^0$ mixing in SM4. First, if we add a sequential fourth up-like quark t', there is produced a new prediction of the mass difference ΔM_{B_s} through the new Wilson coefficients which are related to the fourth generation CKM matrix factors $V^*_{t's}V_{t'b}$. These factors are constrained by the rare decays $B \to X_s\gamma$ in [7]. We find that our results of the prediction of ΔM_{B_s} in SM4 are quite different from that of SM and can satisfy the lower experimental bound in one case of the values $V^*_{t's}V_{t'b}$. In another case, the results are almost the same as in SM. The new effects of the fourth generation show up clearly in the first case. Second, we get the constraint of the fourth generation CKM matrix factor $V^*_{t'b}V_{t'd}$ from the experimental measurement of ΔM_{B_d}. We get one kind of reasonable analytical solution of $V^*_{t'b}V_{t'd}$. This is very small, $-1.0 \times 10^{-4} \le V^*_{t'b}V_{t'd} \le 0.5 \times 10^{-4}$. These results do not contradict the unitarity constraints for the d, b quarks [19].

In Sect. 2, we give the basic formulae for the mass difference $\Delta M_{B_{d,s}}$ in B^0–$\bar{B}^0$ with the sequential fourth generation up-like quark t' in the SM4 model. In Sect. 3, we give the prediction of mass difference ΔM_{B_s} in SM4 and the numerical analysis. Section 4 is devoted to the numerical analysis of the fourth generation CKM matrix factors $V^*_{t'b}V_{t'd}$ from the experimental measurements of the mass difference ΔM_{B_d} in SM4. Finally, in Sect. 5, we give our conclusions.

2 Basic formulae for $\Delta M_{B_{d,s}}$ with t'

$B^0_{d,s}$–$\bar{B}^0_{d,s}$ mixing proceeds to an excellent approximation only through box diagrams with internal top quark exchanges in SM. In SM, the effective Hamiltonian $\mathcal{H}_{\rm eff}$ ($\Delta B = 2$) for $B^0_{d,s}$–$\bar{B}^0_{d,s}$ mixing, relevant for scales $\mu_b = \mathcal{O}(m_b)$, is given by [10]

$$\mathcal{H}^{\Delta B=2}_{\rm eff} = \frac{G_{\rm F}^2}{16\pi^2} M_W^2 (V^*_{tb}V_{tq})^2 S_0(x_t) Q(\Delta B = 2) + \text{h.c.}, \quad (1)$$

where $Q(\Delta B = 2) = (\bar{b}_\alpha q_\alpha)_{V-A}(\bar{b}_\beta q_\beta)_{V-A}$, with $q = d, s$ for $B^0_{d,s}$–$\bar{B}^0_{d,s}$ respectively, and $S_0(x_t)$ is the Wilson coefficient which is taken in the form [10]

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3}{2}\cdot\frac{x_t^3}{(1-x_t)^3}\cdot\ln x, \quad (2)$$

where $x_t = m_t^2/M_W^2$. The mass differences $\Delta M_{d,s}$ can be expressed in terms of the off-diagonal element in the neutral B-meson mass matrix:

$$\Delta M_{d,s} = 2|M_{12}^{d,s}|,$$
$$2m_{B_{d,s}}|M_{12}^{d,s}| = |\langle \bar{B}^0_{d,s}|\mathcal{H}_{\rm eff}(\Delta B = 2)|B^0_{d,s}\rangle|. \quad (3)$$

In SM4, if we add a fourth sequential fourth generation up-like quark t', the above equations would be subjected

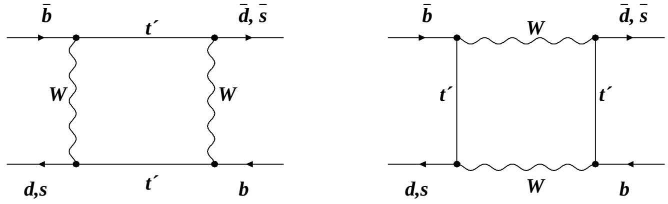

Fig. 1. The additional box diagrams to $B^0_{d,s}$–$\bar{B}^0_{d,s}$ with the fourth up-like quark t'

to some modification. There exist other box diagrams contributed by t' (see Fig. 1), similar to the leading box diagrams in MSSM [15]. The effective Hamiltonian in the standard model, (1), changes to the form [20]

$$\begin{aligned}\mathcal{H}^{\Delta B=2}_{\rm eff} = \frac{G_{\rm F}^2}{16\pi^2} M_W^2 \Big[&\eta_t (V^*_{tb}V_{tq})^2 S_0(x_t) \\ &+ \eta_{t'} (V^*_{t'b}V_{t'q})^2 S_0(x'_t) \\ &+ \eta_{tt'} (V^*_{t'b}V_{t'q})\cdot(V^*_{tb}V_{tq}) S_0(x_t, x'_t)\Big] Q(\Delta B = 2) \\ &+ \text{h.c.} \end{aligned} \quad (4)$$

The mass differences $\Delta M_{d,s}$ in SM4 can be expressed by

$$\begin{aligned}\Delta M_d = \frac{G_{\rm F}^2}{6\pi^2} M_W^2 m_{B_d} (\hat{B}_{B_d}\hat{F}^2_{B_d}) \Big[&\eta_t (V^*_{tb}V_{td})^2 S_0(x_t) \\ &+ \eta_{t'} (V^*_{t'b}V_{t'd})^2 S_0(x_{t'}) \\ &+ \eta_{tt'} (V^*_{t'b}V_{t'd})\cdot(V^*_{tb}V_{td}) S_0(x_t, x_{t'})\Big], \end{aligned} \quad (5)$$

$$\begin{aligned}\Delta M_s = \frac{G_{\rm F}^2}{6\pi^2} M_W^2 m_{B_s} (\hat{B}_{B_s}\hat{F}^2_{B_s}) \Big[&\eta_t (V^*_{tb}V_{ts})^2 S_0(x_t) \\ &+ \eta_{t'} (V^*_{t'b}V_{t's})^2 S_0(x_{t'}) \\ &+ \eta_{tt'} (V^*_{t'b}V_{t's})\cdot(V^*_{tb}V_{ts}) S_0(x_t, x_{t'})\Big], \end{aligned} \quad (6)$$

where $(\hat{B}_{B_s}\hat{F}^2_{B_s}) = \xi_s^2 \cdot (\hat{B}_{B_d}\hat{F}^2_{B_d})$. The new Wilson coefficients $S_0(x_{t'})$ represent the contribution of t', which is like $S_0(x_t)$ in SM in (5), except for exchanging the t' quark, not the t quark. $S_0(x_t, x_{t'})$ represents the contribution of a mixed t–t', which is taken in the form [21]

$$\begin{aligned}S_0(x, y) = x \cdot y \Big[&-\frac{1}{y-x}\frac{1}{4} + \frac{3}{2}\cdot\frac{1}{1-x} \\ &- \frac{3}{4}\cdot\frac{1}{(1-x)^2}\ln x + (y \leftrightarrow x) \\ &- \frac{3}{4}\cdot\frac{1}{(1-x)(1-y)}\Big], \end{aligned} \quad (7)$$

where $x = x_t = m_t^2/M_W^2$, $y = x_{t'} = m_{t'}^2/M_W^2$. The numerical results of $S_0(x_{t'})$ and $S_0(x_t, x_{t'})$ are shown in Table 2.

The short-distance QCD correction factors $\eta_{t'}$ and $\eta_{tt'}$ can be calculated like η_c and η_{ct} in K^0–$\bar{K}^0$ mixing; the NLO values are given in [10, 20], relevant not for the scale $\mathcal{O}(\mu_c)$, but for $\mathcal{O}(\mu_b)$. In leading order, η_t is calculated by

$$\eta_t^0 = [\alpha_{\rm s}(\mu_t)]^{(6/23)}, \quad (8)$$

$$\alpha_{\rm s}(\mu_t) = \alpha_{\rm s}(M_Z)\left[1 + \sum_{n=1}^{\infty}\left(\beta_0 \frac{\alpha_{\rm s}(M_Z)}{2\pi}\text{In}\frac{M_Z}{\mu_t}\right)^n\right],$$

Table 2. The Wilson coefficients $S_0(x_{t'})$ and $S_0(x_t, x_{t'})$ against $m_{t'}$

m_t' (GeV)	50	100	150	200	250	300	350	400	450	500
$S_0(x_{t'})$	0.33	1.07	2.03	3.16	4.44	5.87	7.47	9.23	11.15	13.25
$S_0(x_t, x_{t'})$	0.48	−7.03	−4.94	−5.09	−5.39	−5.87	−5.99	−6.25	−6.49	−6.72
m_t' (GeV)	550	600	650	700	750	800	850	900	950	1000
$S_0(x_{t'})$	15.52	17.97	20.60	23.41	26.40	29.57	32.9336.47	40.96	44.11	
$S_0(x_t, x_{t'})$	−6.92	−7.11	−7.28	−7.44	−7.60	−7.74	−7.87	−7.99	−8.12	−8.23

Table 3. The short-distance QCD factors $\eta_{t'}$, $\eta_{tt'}(=\eta_{t'})$ against $m_{t'}$

m_t' (GeV)	50	100	150	200	250	300	350	400	450	500
$\eta_{t'}$	0.968	0.556	0.499	0.472	0.455	0.443	0.433	0.426	0.420	0.416
m_t' (GeV)	550	600	650	700	750	800	850	900	950	1000
$\eta_{t'}$	0.412	0.408	0.405	0.401	0.399	0.396	0.395	0.393	0.391	0.389

Table 4. Numerical values of the input parameters [11]

$\overline{m}_c(m_c(\text{pole}))$	$1.25 \pm 0.05\,\text{GeV}$	M_W	$80.2\,\text{GeV}$
$\overline{m}_t(m_t(\text{pole}))$	$175\,\text{GeV}$	$\hat{F}_{B_d}\sqrt{\hat{B}_{B_d}}$	$215 \pm 40\,\text{MeV}$
ΔM_{B_d}	$(0.473 \pm 0.016)\,\text{ps}^{-1}$	ξ_s	1.14 ± 0.06
ΔM_{B_s}	$> 14.3\,\text{ps}^{-1}$	G_F	$1.166 \times 10^{-5}\,\text{GeV}^{-2}$

with its numerical value in Table 3. The formulae of the factor $\eta_{t'}$ are similar to the above equation except for exchanging t by t'. For simplicity, we take $\eta_{tt'} = \eta_{t'}$.

The other input parameters necessary in this note are also given (see Table 4).

3 Prediction of ΔM_{B_s} with t'

Experimentally, the mass difference ΔM_{B_s} of the B^0–$\bar{B}^0$ mixing is unclear. It has only a lower bound, $\Delta M_{B_s}^{\text{exp}} > 14.3\,\text{ps}^{-1}$ [11,18]. We have given the calculation formula of ΔM_{B_s} in (6) and the numerical results of the Wilson coefficients S_0 and the QCD correction coefficients η. If we constrain the fourth generation CKM factor $V_{t'b}^* V_{t's}$, we can predict ΔM_{B_s} in the SM4. Fortunately, from our previous paper [7], we have obtained the constraints of $V_{t'b}^* V_{t's}$ from the experimental measurements of $B \to X_s\gamma$. Here, we give only the basic scheme and the final numerical results.

The leading logarithmic calculations can be summarized in a compact form as follows [10]:

$$R_{\text{quark}} = \frac{\text{Br}(B \to X_s\gamma)}{\text{Br}(B \to X_c e\bar{\nu}_e)} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(z)} |C_7^{\text{eff}}(\mu_b)|^2. \tag{9}$$

In the case of four generations there is an additional contribution to $B \to X_s\gamma$ from the virtual exchange of the fourth generation up quark t'. The Wilson coefficients of the dipole operators are given by

$$C_{7,8}^{\text{eff}}(\mu_b) = C_{7,8}^{(\text{SM})\text{eff}}(\mu_b) + \frac{V_{t's}^* V_{t'b}}{V_{ts}^* V_{tb}} C_{7,8}^{(4)\text{eff}}(\mu_b), \tag{10}$$

where $C_{7,8}^{(4)\text{eff}}(\mu_b)$ represent the contributions of t' to the Wilson coefficients, and $V_{t's}^* V_{t'b}$ is the fourth generation CKM matrix factor which we need now. With these Wilson coefficients and the experiment results of the decays of $B \to X_s\gamma$ and $\text{Br}(B \to X_c e\bar{\nu}_e)$ [22,19], we obtain the results of the fourth generation CKM factor $V_{t's}^* V_{t'b}$. There exist two cases:

$$\begin{aligned} V_{t's}^* V_{t'b}^{(+)} &= \left[C_7^{(0)\text{eff}}(\mu_b) - C_7^{(\text{SM})\text{eff}}(\mu_b)\right] \frac{V_{ts}^* V_{tb}}{C_7^{(4)\text{eff}}(\mu_b)} \\ &= \left[\sqrt{\frac{R_{\text{quark}}|V_{cb}|^2 \pi f(z)}{|V_{ts}^* V_{tb}|^2 6\alpha}} - C_7^{(\text{SM})\text{eff}}(\mu_b)\right] \\ &\quad \times \frac{V_{ts}^* V_{tb}}{C_7^{(4)\text{eff}}(\mu_b)}, \end{aligned} \tag{11}$$

$$\begin{aligned} V_{t's}^* V_{t'b}^{(-)} &= \left[-\sqrt{\frac{R_{\text{quark}}|V_{cb}|^2 \pi f(z)}{|V_{ts}^* V_{tb}|^2 6\alpha}} - C_7^{(\text{SM})\text{eff}}(\mu_b)\right] \\ &\quad \times \frac{V_{ts}^* V_{tb}}{C_7^{(4)\text{eff}}(\mu_b)}. \end{aligned} \tag{12}$$

The numerical values are shown in Table 5. With these values, we can give the prediction of ΔM_{B_s} in SM4 in Fig. 2.

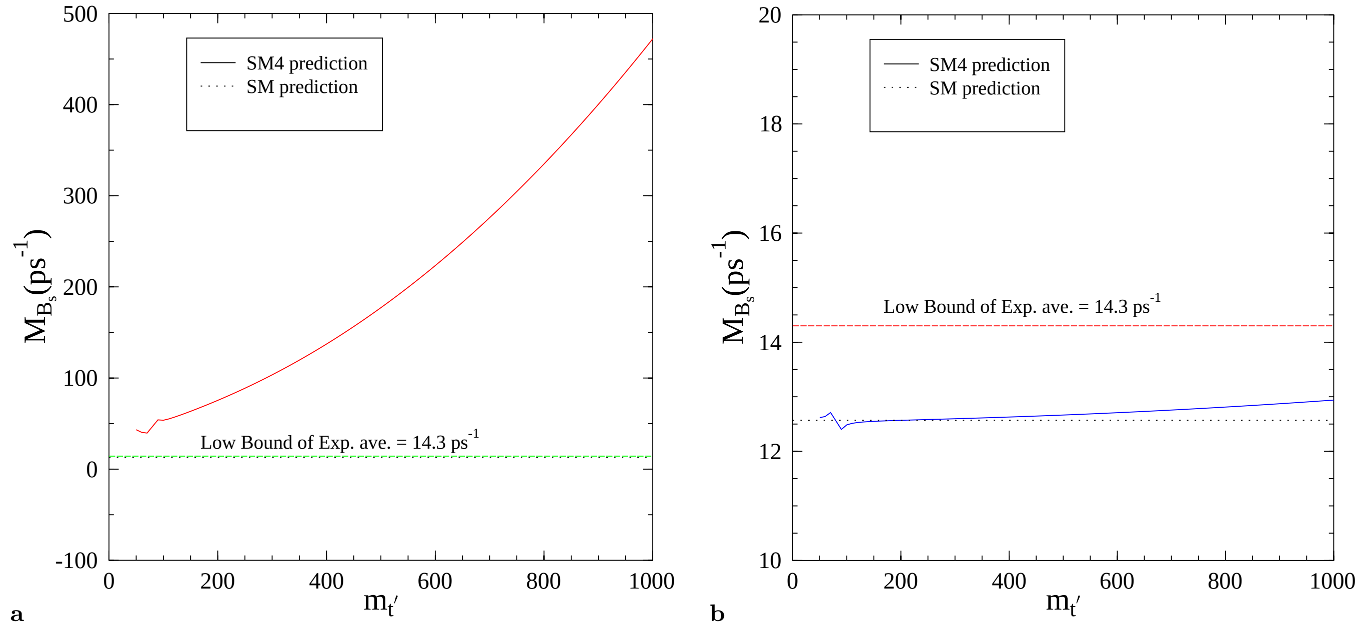

Fig. 2a,b. Prediction of ΔM_{B_s} against $m_{t'}$ in SM4 when $V^*_{t's}V_{t'b}$ takes **a** positive and **b** negative values

Table 5. The values of $V^*_{t's} \cdot V_{t'b}$ due to the masses of t' for $\mathrm{Br}(B \to X_s\gamma) = 2.66 \times 10^{-4}$

$m_{t'}$ (GeV)	50	100	150	200	250	300	350
$V^*_{t's}V^{(+)}_{t'b}/10^{-2}$	−11.591	−9.259	−8.126	−7.501	−7.116	−6.861	−6.580
$V^*_{t's}V^{(-)}_{t'b}/10^{-3}$	3.564	2.850	2.502	2.309	2.191	2.113	2.205

$m_{t'}$ (GeV)	400	500	600	700	800	900	1000
$V^*_{t's}V^{(+)}_{t'b}/10^{-2}$	−6.548	−6.369	−6.255	−6.178	−6.123	−6.082	−6.051
$V^*_{t's}V^{(-)}_{t'b}/10^{-3}$	2.016	1.961	1.926	1.902	1.885	1.872	1.863

The mass differences ΔM_{B_s} in these two cases are shown in Fig. 2a,b respectively. In the second case, in which $V^*_{t's}V_{t'b}$ takes a positive value, i.e. $(V^*_{t's}V^{-}_{t'b})$, the curve of ΔM_{B_s} against $m_{t'}$ almost overlaps with that of SM. That is, the results in SM4 are the same as that in SM. In this case, there does not show up new effects of t'. The mass difference ΔM_{B_s} is still unclear. We cannot obtain information on the existence of the fourth generation from ΔM_{B_s}, nor can we exclude it. The reason is that, from Table 5, although the values of $V^*_{t's}V^{(-)}_{t'b}$ are positive, they are of order 10^{-3} and so are very small. The values of $V^*_{ts}V_{tb}$ are about ten times larger than those of $V^*_{ts} = 0.038$, $V_{tb} = 0.9995$ [19]. Furthermore, the last two terms on $m_{t'}$ in (6) are approximately of the same order. The contributions of them counteract each other.

But in the second case the values of $V^*_{t's}V_{t'b}$ are negative, i.e. $(V^*_{t's}V^{(-)}_{t'b})$. The curve of ΔM_{B_s} is quite different from that of the SM. This can clearly be seen from Fig. 2b. The enhancement of ΔM_{B_s} increases rapidly with increasing of the t' quark mass. In this case, the fourth generation effects are shown clearly. The reason is that $V^*_{t's}V^{(+)}_{t'b}$ is 2–3 times larger than $V^*_{ts}\dot{V}_{tb}$ so that the last two terms on $m_{t'}$ in (6) becomes important and it strongly depends on the t' mass. Thus, the effect of the fourth generation is significant. Meanwhile, the prediction of ΔM_{B_s} in SM4 can satisfy the experimental lower bound of $\Delta M_{B_s} \geq 14.3\,\mathrm{ps}^{-1}$. So the sequential fourth generation model could be one of the ways of searching new physics on ΔM_{B_s}. If $V^*_{t's}V_{t'b}$ is chosen in this case, the mass difference ΔM_{B_s} in B^0–$\bar{B}^0$ mixing can give a good probe of the existence of the fourth generation.

4 Constraints of the fourth generation CKM factor $V^*_{t'b}V_{t'd}$ from experimental measurements of ΔM_{B_d}

Unlike ΔM_{B_s}, the mass difference ΔM_{B_d} of B^0_d–$\bar{B}^0_d$ mixing is experimentally clear, $\Delta M^{\mathrm{exp}}_{B_d} = 0.473 \pm 0.016\,\mathrm{ps}^{-1}$ [11]. We can get the constraints of the fourth generation CKM factor $V^*_{t'b}V_{t'd}$ from the present experimental value of ΔM_{B_d}.

We change the form of (5) as a quadratic equation of $V^*_{t'b}V_{t'd}$. By solving it, we can get two analytical solutions $V^*_{t'd}V^{(1)}_{t'b}$ (in which the absolute value is large) and $V^*_{t'd}V^{(2)}_{t'b}$ (in which the absolute value is small), just like the other

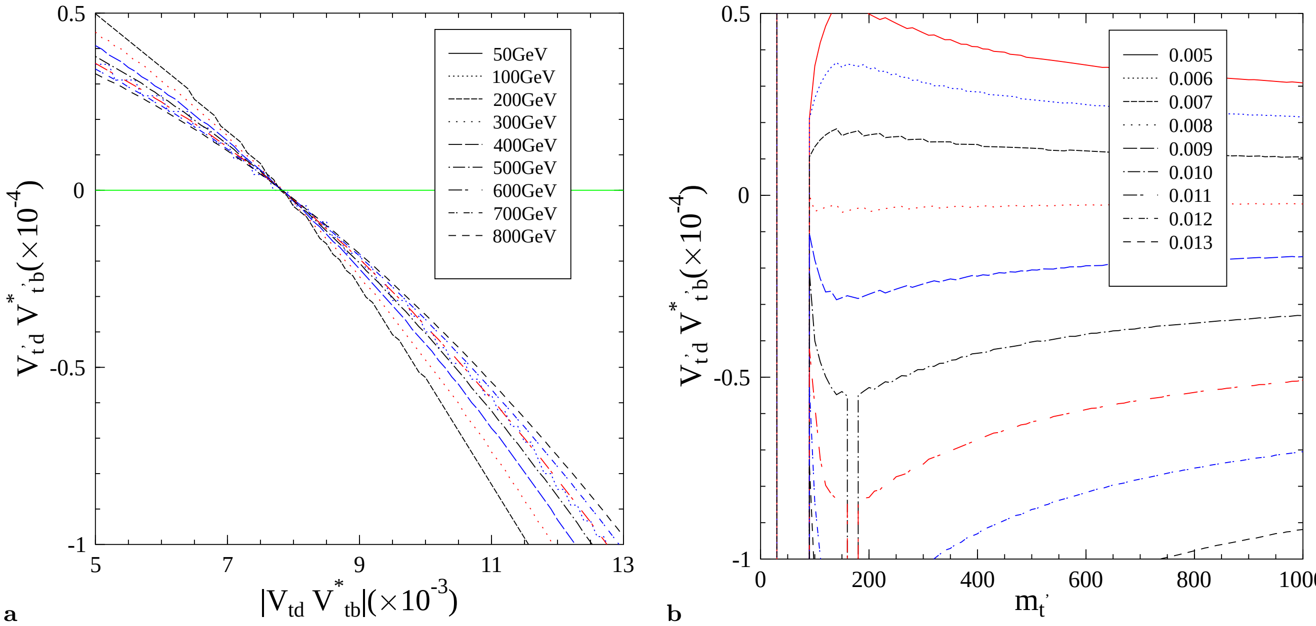

Fig. 3a,b. Constraint of the fourth generation CKM factor $V^*_{t'd}V_{t'b}$ to **a** $|V_{td}V^*_{tb}|$ with $m_{t'}$ ranging from 50 GeV to 800 GeV, **b** to $m_{t'}$ with $|V_{td}V^*_{tb}|$ ranging from 0.005 to 0.013

fourth generation CKM matrix factor $V^*_{t's}V^{(\pm)}_{t'b}$ [8] in the last section.

However, experimentally, it is not accurate for the measurement of the CKM matrix element V_{td} [10,19]. So, we have to search other ways to solve this difficulty. Fortunately, we have the CKM unitarity triangle [23], i.e. the graphic representation of the unitarity relation for d, b quarks, which come from the orthogonality condition on the first and third row of $V_{\rm CKM}$,

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0. \qquad (13)$$

From the above equation, we can give the constraints of $V_{td}V^*_{tb}$ [24],

$$0.005 \leq |V_{td}V^*_{tb}| \leq 0.013. \qquad (14)$$

Then we can give the final results as shown in Fig. 3a,b.

We must announce that Fig. 3 shows the curves with $V^*_{t'd}V^{(2)}_{t'b}$ (the absolute value is the small one) only. This is because the absolute value of $V^*_{t'd}V^{(1)}_{t'b}$ is generally larger than 1. This is in contradiction with the unitarity of the CKM matrix. So we do not study this solution. From Fig. 3, we find that all curves are in the range from -1×10^{-4} to 0.5×10^{-4} when we are considering the constraint of $V_{td}V^*_{tb}$. That is to say, the absolute value of $V^*_{t'd}V_{t'b}$ is about $\sim 10^{-4}$, two orders smaller than the sum of the first three ones on the left of (14). Considering that the data of the CKM matrix are not very accurate, the values of $V^*_{t'd}V^{(2)}_{t'b}$ are safely in the error range of the sum of these first three terms and do not contradict the CKM matrix unitarity constraints.

5 Conclusion

We have investigated the mass differences $\Delta M_{B_{d,s}}$ in $B^0_{d,s}$–$\bar{B}^0_{d,s}$ mixing with a new up-like quark t' in a sequential fourth generation model. The fourth generation quark t' will obviously give new effects on the mass difference $\Delta M_{B_{d,s}}$ if it really exists. At least, the present experimental statue of $\Delta M_{B_{d,s}}$ could not exclude the existence of the fourth generation. Furthermore, the progress of theoretical calculation and experimental measurement of $\Delta M_{B_{d,s}}$ could provide a strong test of the existence of the fourth generation. In other words, indicating one of the directions beyond the SM, $\Delta M_{B_{d,s}}$ could provide possible tests of the fourth generation and signals of new physics.

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References

1. G.S. Abrams et al., Mark II Collab., Phys. Rev. Lett. **63**, 2173 (1989); B. Advera et al., L3 Collab., Phys. Lett. B **231**, 509 (1989); I. Decamp et al., OPAL Collab., ibid., **231**, 519 (1989); M.Z. Akrawy et al., DELPHI Collab., ibid., **231**, 539 (1989); C. Caso et al. (Particle Data Group), Eur. Phys. J. C **3**, 1 (1998)
2. Z. Berezhiani, E, Nardi, Phys. Rev. D **52**, 3087 (1995); C.T. Hill, E.A. Paschos, Phys. Lett. B **241**, 96 (1990)
3. Y. Nir, D. Silverman, Phys. Rev. D **42**, 1477 (1990); W-S, Choong, D. Silverman, Phys. Rev. D **49**, 2322 (1994); L.T. Handoko, hep-ph/9708447

4. V. Barger, Y.B. Dai, K. Whisnant, B.L. Young, hep-ph/9901380; R.N. Mohapatra, hep-ph/9702229; S. Mohanty, D.P. Roy, U. Sarkar, hep- ph/9810309; S.C. Gibbons, et al., Phys. Lett. B **430**, 296 (1998); V. Barger, K. Whisnant, T.J. Weiler, Phys. Lett. B **427**, 97 (1998); V. Barger, S. Pakvasa, T.J. Weiler, K. Whisnant, Phys. Rev. D **58**, 093016 (1998)
5. J.F. Gunion, Douglas W. McKay, H. Pois, Phys. Lett. B **334**, 339 (1994); Phys. Rev. D **51**, 201 (1995)
6. J. Swain, L. Taylor, hep-ph/9712383; R.N. Mohapatra, X. Zhang, hep-ph/9301286; V. Novikov, hep-ph/9606318; L.L. Smith, D. Jain, hep-ph/9501294; K.C. Chou, Y.L. wu, Y.B. Xie, Chinese Phys. Lett. **1**, 2 (1984); A. Datta, hep-ph/9411435; T. Yoshikawa, Prog. Theor. Phys. **96**, 269 (1996); D. Grosser, Phys. Lett. B **86**, 301 (1979); C.D. Froggatt, H.B. Nielsen, D.J. Smith, Z. Phys. C **73**, 333 (1997); S. Dimopoulos, Phys. Lett. B **129**, 417 (1983); LEP1.5 Collab., J. Nachtman, hep-ex/960615
7. C.S. Huang, W.J. Huo, Y.L. Wu, Mod. Phys. Lett. A **14**, 2453 (1999)
8. C.S. Huang, W.J. Huo, Y.L. Wu, hep-ph/0005227
9. DØ Collab., S. Abachi et al., Phys. Rev. Lett. **78**, 3818 (1997)
10. A.J. Buras, hep-ph/9806471
11. A. Ali, D. London, hep-ph/0002167
12. H. Alberecht et al., Phys. Lett. B **192**, 245 (1987); M. Artuso et al., Phys. Rev. Lett. **62**, 2233 (1989)
13. J.H. Christenson, J.W. Cronin, V.L. Fitch, R. Turlay, Phys. Rev. Lett. **13**, 128 (1964)
14. Y. Grossman, M. Worah, Phys. Lett. B **395**, 241 (1997); M. Worah, hep-ph/9711265; N.G. Deshpande, B. Dutta, S. Oh, Phys. Rev. Lett. **77**, 4499 (1996); M. Ciuchni et al., Phys. Rev. Lett. **79**, 978 (1997); D. London, A. Soni, Phys. Lett. B **407**, 61 (1997); A. Abd El-Hady, G. Valencia, Phys. Lett. B **414**, 173 (1997); J.P. Silva, L. Wolfenstein, Phys. Rev. D **55**, 5331 (1997); A.I. Sanda, Z.Z. Xing, Phys. Rev. D **56**, 6866 (1997); S.A. Abel, W.N. Cottingham, I.B. Whittingham, Phys. Rev. D **58**, 073006 (1998)
15. I. Hinchliffe, N. Kersting, hep-ph/0003090
16. G. Barenboim, J. Bernabeu, J. Matias, M. Raidal, hep-ph/9901265; Y. Grossman, Phys. Lett. B **380**, 99 (1996); Z.Z. Xing, Eur. Phys. J. C **4**, 283 (1998); Y. Grossman, Y. Nir, R. Rattazzi, hep-ph/9701231
17. G. Blaylock, http://www.cern.ch/LEPBOSC/
18. S. Willocq, hep-ex/0002059
19. C. Caso et al. (Particle Data Group), Eur. Phys. J. C **3**, 1 (1998)
20. S. Herrich, U. Nierste, hep-ph/9604330; hep-ph/9310311; S. Herrich, hep-ph/9609376
21. J.F. Donoghue, E. Golowich, B.R. Holstein, Dynamics of the standard model (Cambridge University Press, New York 1992)
22. M.S. Alam, Phys. Rev. Lett. **74**, 2885 (1995)
23. A. Ali, hep-ph/9606324; hep-ph/9612262
24. G. Barenboim, G. Eyal, Y. Nir, hep-ph/9905397